Intelligent Digital Redesign: A Fuzzy Output Case

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Abstract
An intelligent digital redesign technique (IDR) for the observer-based output feedback Takagi-Sugeno (T-S) fuzzy control system with fuzzy outputs is developed. The considered IDR condition is cubicly parameterized as convex minimization problems of the norm distances between linear operators to be matched. The stability condition is easily embedded and the separations principle is explicitly shown.

Key Words: Fuzzy controller, Intelligent digital redesign, TSK fuzzy system

1. Introduction

It has been noted that the digital redesign schemes basically work only for a class of linear systems [1]. For that reason, it has been highly demanded to develop some intelligent digital redesign methodology for complex nonlinear systems, in which the first attempt was made by Joo et al. [2]. They synergistically merged both the Takagi-Sugeno (T-S) fuzzy-model-based control and the digital redesign technique for a class of nonlinear systems. Chang et al. extended the intelligent digital redesign to uncertain T-S fuzzy systems [3] and elaborated it [4]. However, until now, no tractable method for IDR tackling on the observer-based output-feedback T-S fuzzy system with fuzzy outputs has been proposed.

They remain yet to be theoretically challenging issues in IDR and thereby must be fully tackled.

Motivated by the above observations, this paper aims at developing IDR for the observer-based output-feedback T-S fuzzy control system with fuzzy outputs. To resolve the problems above stated, we propose an alternative way-convex optimization-based IDR. The main contribution of this paper is to derive sufficient conditions of IDR in terms of linear matrix inequalities. The stability condition is naturally incorporated with ease. The separation principle is also explicitly shown.

The rest of this paper is organized as follows: Section 2 briefly reviews T-S fuzzy systems both continuous and discrete-time cases. In Section 3, a new IDR method is proposed for observer-based output-feedback T-S fuzzy control systems. This paper concludes with Section 4.

2. Preliminaries

Consider a TS fuzzy system in which the ith rule is formulated in the following form:

\[ R^i: \text{IF } x(t) \text{ is about } \Gamma^i_0, \ldots, x(t) \text{ is about } \Gamma^i_n \]

\[ \text{THEN } \dot{x}_i(t) = A x(t) + B_i u(t) \]

\[ y(t) = C_i x(t) \]

where \( x(t) \in \mathbb{R}^n \) is the state vector, \( u(t) \in \mathbb{R}^m \) is the control input vector. The subscript `c' means the analog control, while the subscript `d' will denote the digital control in the sequel. \( R^i \) denotes the ith fuzzy inference rule. \( z_i(t) \) is the premise variable, \( \Gamma^i_k, k = I_{Q_k}, k = I_{N_k} \) is the fuzzy set of the ith premise variable in the ith fuzzy inference rule. Using the center-averaging defuzzification, product inference, and singleton fuzzifier, the global dynamics of this T-S fuzzy system (1) is described by

\[ \dot{x}(t) = \sum_{i=1}^{m} \theta_i(z(t)) (A x(t) + B_i u(t)) \]
\[ x_c(t) = \sum_{i} C_i x_i(t) \]

in which  \( \omega_i(z_i(t)) = \prod_{i=1}^{n} \Gamma_i(z_i(t)) \)

\[ \theta_i(z(t)) = \omega_i(z(t))/\sum_i \omega_i(z(t)) \]

and \( \Gamma_i(z_i(t)) \) is the membership value of the \( i \)th premise variable \( z_i(t) \) in \( \Gamma_i \).

Throughout this paper, a well-constructed continuous-time observer-based fuzzy-model-based control law is assumed to be pre-designed, which will be used in redesigning the digital control law. In real control problems, we cannot always observe all the states of a system. Hence a fuzzy-model-based observer is introduced as follows:

\[ R^i_1: \text{IF } z_i(t) \text{ is about } \Gamma^i_1 \text{ THEN } x_c(t) = P_i \varphi_i A_i x_i(t) + B_i u_i(t) \]

The defuzzified output of the controller rules is given by

\[ x_c(t) = \sum_i \theta_i(z_i(t))(A_i x_i(t) + B_i u_i(t)) \]  

\[ + L_i y_i(t) \]

The controller rule is of the following form:

\[ R^i_k: \text{IF } z_i(t) \text{ is about } \Gamma^i_k \text{ THEN } u_i(t) = K_i \varphi_i x_c(t) + O_i y_i(kT) \]

for \( i = \{1, 2, \ldots, n\} \) and \( k \) is the digital control gain matrix to be determined for the \( i \)th rule, and the overall control law is given by

\[ u_i(t) = \sum_i \theta_i(z_i(t))(K_i \varphi_i x_c(t) + O_i y_i(kT)) \]

for \( i = \{1, 2, \ldots, n\} \).

The objective is to find gain matrices for digital controller and observers in (7) and (6) from the analog gain matrices in (3) and (2), so that the closed-loop state \( x_i(t) \) in (5) with (7) can closely match the closed-loop state \( x_c(t) \) in (4) at all sampling time instants \( t = kT, k \in Z^+ \). Thus it is more convenient to convert the TS fuzzy system into discrete-time version for derivation of the state matching condition.

**Theorem 1:** The pointwise dynamical behavior of the TS fuzzy system (5) can be efficiently approximated by

\[ x_i(kT+T) \approx \sum_i \theta_i(z_i(kT))(G x(t) + H_i u_i(kT)) \]

**Proof:** See the reference [5].

**Remark 1:** The discretized TS fuzzy system (8) contains the discretization error with the order of \( O(T^2) \), which is tolerable under the choice of a sufficiently small sampling period, and vanishes as \( T \) approaches zero. Notice that the error induced in this discretization procedure is smaller than the first-order truncated Taylor series expansion of (5).

\[ x_c(t) = \sum_i \theta_i(z_i(t))(A_i x_i(t) + B_i u_i(t)) \]

\[ \sum_i \theta_i(z_i(t))(G_i x_i(t) + H_i u_i(t)) + L_i y_i(t) \]

where \( G_i = \Theta_i (A_i - L_i C_i) \) and \( u_i(t) = u_i(kT) \) is the piecewise-constant control input vector to be determined, in the time interval \( [kT, kT+T] \), and \( T > 0 \) is a sampling period. For the digital control of the continuous-time TS fuzzy system, the digital fuzzy-model-based controller is employed. Let the fuzzy rule of the digital control law for the system (5) take the following form:

\[ R^i_1: \text{IF } z_i(t) \text{ is about } \Gamma^i_1 \text{ THEN } u_i(t) = K_i \varphi_i x_c(t) + O_i y_i(kT) \]

\[ \text{for } i = \{kT, kT+T\} \], where \( K_i \) and \( O_i \) are the digital control gain matrix to be determined for the \( i \)th rule, and the overall control law is given by

\[ u_i(t) = \sum_i \theta_i(z_i(t))(K_i \varphi_i x_c(t) + O_i y_i(kT)) \]

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\[ \sum_i \theta_i(z_i(t))(G_i x_i(t) + H_i u_i(t)) + L_i y_i(t) \]

where \( G_i = \Theta_i (A_i - L_i C_i) \) and \( u_i(t) = u_i(kT) \) is the piecewise-constant control input vector to be determined, in the time interval \( [kT, kT+T] \), and \( T > 0 \) is a sampling period. For the digital control of the continuous-time TS fuzzy system, the digital fuzzy-model-based controller is employed. Let the fuzzy rule of the digital control law for the system (5) take the following form:

\[ \text{R}^i_1: \text{IF } z_i(t) \text{ is about } \Gamma^i_1 \text{ THEN } u_i(t) = K_i \varphi_i x_c(t) + O_i y_i(kT) \]

\[ \text{for } i = \{kT, kT+T\} \], where \( K_i \) and \( O_i \) are the digital control gain matrix to be determined for the \( i \)th rule, and the overall control law is given by

\[ u_i(t) = \sum_i \theta_i(z_i(t))(K_i \varphi_i x_c(t) + O_i y_i(kT)) \]

\[ \text{for } i = \{kT, kT+T\} \]

The objective is to find gain matrices for digital controller and observers in (7) and (6) from the analog gain matrices in (3) and (2), so that the closed-loop state \( x_i(t) \) in (5) with (7) can closely match the closed-loop state \( x_c(t) \) in (4) at all sampling time instants \( t = kT, k \in Z^+ \). Thus it is more convenient to convert the TS fuzzy system into discrete-time version for derivation of the state matching condition.

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**Proof:** See the reference [5].

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\[
    x_d(kT + T) \approx \sum_{i=1}^{N_d} \sum_{j=1}^{N_d} \delta_{ij}(z(kT)) \theta_{ij}(z(kT)) \theta_{ij}(z(kT)) \times \begin{bmatrix}
    G_i + H_i K_i + H_i O_i C_j & L_i C_j \\
    G_i - L_i C_j & 0
\end{bmatrix} x(kT)
\]

where \( x_d(kT) \) is the state vector of the closed-loop system.

Corollary 1: The pointwise dynamical behavior of the continuous-time closed-loop TS fuzzy system (9) can also be approximately discretized as

\[
    x(kT + T) \approx \sum_{i=1}^{N_d} \sum_{j=1}^{N_d} \delta_{ij}(z(kT)) \theta_{ij}(z(kT)) \Phi \circ \phi_n(kT)
\]

where

\[
    \Phi = \exp \left( \begin{bmatrix}
        A_i + B_i K_i & L_i C_j \\
        A_i - L_i C_j & 0
    \end{bmatrix} \right)
\]

for \((i,j) \in I_0 \times I_0\), where \( \phi_n(kT) \) is the state of the observer.

Proof: It can be straightforwardly proved by Theorem 1.

3.2 Main Results

The IDR problem can be formulated as follows:

Problem 1: Given the well-designed gain matrices \( K_i \) and \( L_i \) for (3) and (2), find \( K_i \) and \( O_i \) for (7), and \( L_i \) for (6) such that the following are satisfied:

1. The state \( x(kT) \) of (9) matches the state \( x(kT) \) of (10) at every sampling time instance \( t = kT \), as closely as possible.

2. The digitally controlled system is asymptotically stable.

Theorem 2: Suppose (9) is asymptotically stable in the sense of Lyapunov; then the zero equilibrium points \( x_d(0) = [0] \) and \( e_d(kT) = [0] \) of the hybrid control system that consists of (5), (6), and (7) are also asymptotically stable.

Proof: The proof is omitted due to lack of space.

Hence, Problem 1 can be equivalently converted as follows:

Problem 2: Given the well-designed gain matrices \( K_i \) and \( L_i \) for (3) and (2), find \( K_i \) and \( O_i \) for (7), and \( L_i \) for (6) such that the following are satisfied:

1. Minimize \( \gamma_1 \) and \( \gamma_2 \) over \( K_i \) and \( O_i \), subject to \( \| \phi_1 \| - G_i - H_i K_i - H_i O_i C_i \| < \gamma_1 \), \( \| \phi_2 \| + H_i K_i \| < \gamma_2 \), and \( \| \phi_3 \| - G_i + L_i C_i \| < \gamma_2 \) in the sense of the spectral norm measure.

(2) The discretized closed-loop system (9) is asymptotically stable in the sense of Lyapunov criterion.

Remark 2: Equation (10) can be viewed as a convex combination of quadratically parameterized sub-closed-loop systems. Similarly (9) is also a convex combination of cubically parameterized sub-closed-loop systems. It yields the minimization problem of the norm distances between \( q^2 \) or \( q^3 \) of \( G_i + H_i K_i \) and \( H_i O_i C_i \) sub-closed-loop systems. It is necessary to examine whether the upper bound \( \gamma_1 \) is guaranteed or not in this case.

Theorem 3: Suppose \( \| \phi_1 \| - G_i - H_i O_i C_i \| < \gamma_1 \) for all \((i,j,b) \in I_0 \times I_0 \times I_0\), then the following holds:

\[
    \| \phi_2 \| + H_i K_i \| < \gamma_2 \}
\]

Proof: The proof is omitted due to lack of space.

Now, the main result is summarized as follows:

Theorem 4: If there exist symmetric positive definite matrices \( P, Q, X \), matrices \( K_i, O_i, N_i \), with appropriate dimensions, and possibly small positive scalars \( \gamma_1 \) and \( \gamma_2 \) such that the following two generalized eigenvalue problems (GEVPs) have solutions

GEVP 1:

\[
    \begin{bmatrix}
        -\gamma_1 P & \star \\
        -\gamma_1 Q & \star
    \end{bmatrix} < 0
\]

GEVP 2:

\[
    \begin{bmatrix}
        -P & \star \\
        -Q & \star
    \end{bmatrix} < 0
\]

\[
    \begin{bmatrix}
        -X & \star \\
        -I & -P
    \end{bmatrix} < 0
\]

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GEVP 2:

\[
\text{minimize } \quad Q, N_i \quad \gamma_2 \quad \text{subject to} \quad \begin{bmatrix} \hat{\phi} & \hat{\phi} \\ \phi & \phi \end{bmatrix}^T \begin{bmatrix} -\gamma_2 Q & \star \\ \star & -\gamma_2 J \end{bmatrix} \begin{bmatrix} \phi \\ \phi \end{bmatrix} < 0 \]
\[
\begin{bmatrix} G_i & C_i & N_i \\ C_i & N_i & Q \end{bmatrix} \begin{bmatrix} \phi \\ \phi \end{bmatrix} < Q < 0
\]

then, the state \( x(kT) \) of the discretized version (8) of the controlled system via the redesigned digital fuzzy-mode-based controller (7) closely matches the state \( x(kT) \) of the discretized version of the analogously controlled system (10). Furthermore, the discretized system (8) is asymptotically stabilizable in the sense of Lyapunov stability criterion, where star denotes the transposed element in symmetric positions.

Proof: The proof is omitted due to lack of space.

Remark 3: It is important to address that, since the searching variables \( P, K_i, Q_i, X \) for the digital controller in GEVP 1 and \( Q, N_i \) for the discrete observer in GEVP 2 of Theorem 2 are not coupled, the IDR for the digital controller (7) and the discrete observer (6) can be performed independently, which indicates that the separation principle holds for IDR of the observer-based output-feedback fuzzy-model-based controller.

4. Concluding Remarks

In this paper, a new IDR has been proposed for the observer-based output-feedback fuzzy-model-based controller. The developed technique formulated the given IDR problem as constrained convex optimization problems so that the powerful and flexible numerical algorithms can be utilized. The flexibility of the LMIs enables one to incorporate the stability of the redesigned system into the IDR algorithm. The separation principle was clearly shown.

References


