**Robust Observer-based Fuzzy Control for Variable Speed Wind Power System: LMI Approach**

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**Abstract:** In this paper, the output-feedback control for stabilizing the uncertain nonlinear system is proposed. For achieving the robust stability, we deal with the parametric uncertainties of the concerned system which is based on the Takagi-Sugeno (T-S) fuzzy model. Also, we derive the observer-based models preserving the property and structure of the uncertainties. The sufficient conditions for output feedback stabilizing controller designs are given in terms of solutions to a set of linear matrix inequalities (LMIs). The simulation results for variable speed wind power (VSWP) system are demonstrated to visualize the feasibility of the proposed method.

**Keywords:** Linear matrix inequalities (LMIs), observer-based control, parametric uncertainties, Takagi-Sugeno (T-S) fuzzy model, variable speed wind power (VSWP) system.

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**1. INTRODUCTION**

Since in many real plants, especially renewable power systems, state variables are often unavailable, output feedback control is necessary and has caused some interest. Also, practical systems are commonly required to have good robustness so that robust control problems become important research topic in control theory. The parametric uncertainty is a principal factor responsible for the degraded stability and the performance of an uncertain control system. In fact, in many cases it is very difficult, if not impossible, to obtain the accurate values of some system parameters. This is due to the inaccurate measurement, inaccessibility to the system parameters, or on-line variation of the parameters.

As a result, there have been many researches to deal with the output feedback control problem of uncertain systems. The references [4,6,11,12] have studied various robust control problems for linear continuous or discrete systems with observer. Besides linear systems, the robust control and output feedback control for complex nonlinear systems have also been lively discussions. Among them, fuzzy control approach have attracted investors' attention thanks to its usefulness, for example, Tanaka et al. proposed the fuzzy observer design for nonlinear control systems [10]. The robustness in Takagi-Sugeno (T-S) fuzzy control has been extensively studied for some control techniques [1-3,7]. However, except [2], there have been few studies of the output feedback control approach for the fuzzy system with parametric uncertainties.

In [2], Tong et al. dealt with the observer-based control for uncertain nonlinear system which was represented by linear matrix inequality (LMI) formats. However, the synthesis conditions in [2] are invalid because of the incorrect use of a property that resulted in neglecting certain terms so that the obtained conditions are not LMI but bilinear matrix inequality (BMI) formulation [3]. This is very important point because the algorithm in [2] is nonconvex so that iterative linear matrix inequality (ILMI) approaches should be required. As a result, the separation principle is not guaranteed for the obtained conditions [3]. The sufficient LMI conditions have the advantage of being linear and, hence, easily tractable by standard optimization techniques [6]. Although the new observer-based control design method was studied in [5], this approach did not deal with the parametric uncertainties problem but the disturbance one. Therefore, to our best knowledge, there are no studies for observer-based wind power system control with parametric uncertainties by LMI approach.

Motivated by the above observations, this paper presents a robust control method for both controller and observer designs for a general nonlinear system with parametric uncertainties. The robust stabilization problems are derived for a class of nonlinear system with time-varying but norm-bounded parametric uncertainties even though their state variables are not available for measurement. Also, we derive the observer-based models of the fuzzy control preserving the property and structure of the uncertainties. Its constructive conditions are provided in the LMI formats, and therefore easily tractable by the convex optimization techniques. The
obtained methods are applied to the variable speed wind power (VSWP) system which is constructed as the T-S fuzzy model.

The paper is organized as follows: Section 2 reviews T-S fuzzy models and fuzzy observers. The state feedback and output feedback controller designs for robust stabilization of T-S fuzzy systems with parametric uncertainties are presented in Section 3. Section 4 shows a design example of VSWP system and simulation results. Finally, conclusion is given in Section 5.

2. PRELIMINARIES

Consider a nonlinear system of the following form:

\[
\begin{align*}
\dot{x}(t) &= f(x(t), u(t)) \\
y(t) &= h(x(t)),
\end{align*}
\]

where \(x(t) \in \mathbb{R}^n\) constitutes the state vector; \(u(t) \in \mathbb{R}^n\) is the control input; \(y(t) \in \mathbb{R}^p\) is the output. We define compact sets for \(x(t)\) and \(u(t)\) such that \(\varphi_x = \{x | \|x\| \leq \Delta_x\} \subset \mathbb{R}^n\), \(\varphi_u = \{u | \|u\| \leq \Delta_u\} \subset \mathbb{R}^m\) for some \(\Delta_x, \Delta_u \in \mathbb{R}_{>0}\) and \(\Delta_u \in \mathbb{R}_{>0}\). Within the scope of the above bound, it is possible to represent the nonlinear system as the T-S fuzzy rules without any approximation on the T-S fuzzy rules (1) can be represented as the following T-S fuzzy rules:

\[
R^i: \text{IF } z_i(t) \text{ is about } \Gamma^i_1 \text{ and... and } z_p(t) \text{ is about } \Gamma^i_p \text{ THEN } \begin{align*}
\dot{x}(t) &= (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) \\
y(t) &= C_i x(t),
\end{align*}
\]

where \(R_i, i \in \Pi_i = \{1,2,\cdots,q\}\), denotes the \(i\) th fuzzy rule, \(z_i(t), h \in \Pi^p = \{1,2,\cdots,p\}\), is the \(h\) th premise variable, \(\Gamma^i_1, (i, h) \in \Pi_i \times \Pi^p\), is the fuzzy set of \(z_i(t)\) in \(R_i\); \(A_i\) and \(B_i\) are compact matrices with appropriate dimensions, and \(\Delta A_i\) and \(\Delta B_i\) are unknown matrices with appropriate dimensions which represent the parametric uncertainties. Using the singleton fuzzifier, product inference engine, and center-average defuzzification, (2) is inferred as

\[
\begin{align*}
\hat{x}(t) &= \sum_{i=1}^{q} \hat{\theta}_i(z(t))(A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) \\
y(t) &= \sum_{i=1}^{q} \hat{\theta}_i(z(t))C_i x(t),
\end{align*}
\]

where \(\hat{\theta}_i(z(t)) = w_i(z(t)) / \sum_{i=1}^{q} w_i(z(t))\) and \(w_i(z(t)) = \prod_{h=1}^{p} \mu_{i_h}(z_h(t))\) is the membership function of \(z_h(t)\) on the compact set \(U_{z_h(t)}\).

To estimate the state, the following full-order observer is utilized:

\[
\begin{align*}
\hat{x}(t) &= \sum_{i=1}^{q} \theta_i(z(t))(A_i x(t) + B_i u(t) + L_i(y(t) - \hat{y}(t))) \\
\hat{y}(t) &= \sum_{i=1}^{q} \theta_i(z(t))C_i \hat{x}(t),
\end{align*}
\]

where \(\hat{x}(t) \in \mathbb{R}^n\) is the estimation of \(x(t)\) and \(L_i\) is the observer gain matrix. In this paper, the following fuzzy rule for the controller is employed:

\[
R^i : \text{IF } z_i(t) \text{ is about } \Gamma^i_1 \text{ and... and } z_p(t) \text{ is about } \Gamma^i_p \text{ THEN } u(t) = -K_i \hat{x}(t),
\]

whose defuzzified output is

\[
u(t) = -\sum_{i=1}^{q} \theta_i(z(t))K_i \hat{x}(t),
\]

where the control input \(u(t)\) is designed by parallel distributed compensation (PDC) approach [10]. Further, we assume that the uncertainties \(\Delta A_i\) and \(\Delta B_i\) have the following structure.

**Assumption 1:** We assume that \(\Delta A_i\) and \(\Delta B_i\) can be described as follows:

\[
[\Delta A_i, \Delta B_i] = D_i F_i(t)[E_{1i}, E_{2i}],
\]

where \(D_i, E_{1i}\) and \(E_{2i}\) are known real constant matrices of compatible dimensions, and \(F_i(t)\) is an unknown matrix function with Lebesgue-measurable elements and with \(F_i^T(t)F_i(t) \leq I\).

There are fruitful research works focusing on the design of the output feedback fuzzy controller [10] and robust fuzzy controller [1-3,5,7]. However, the studies of output feedback control for real wind plants have not been sufficiently achieved. Although reference [2] tried to concern that problem, this approach did not guarantee the LMI, but BMI formulation. Our paper aims at a robust fuzzy control method for a general nonlinear system with parametric uncertainties.

3. NEW ROBUST OBSERVER-BASED FUZZY CONTROL METHODOLOGY

Let the state estimation error be \(e(t) = x(t) - \tilde{x}(t)\), then the closed-loop system is described as follows:

\[
\begin{align*}
\dot{x}(t) &= \sum_{i=1}^{q} \theta_i(z(t))(A_i x(t) + B_i u(t) + L_i(y(t) - \hat{y}(t))) \\
&\quad \times x(t) + \sum_{i=1}^{q} \theta_i(z(t))\theta_j(z(t))(A_i - B_i K_j)\varepsilon(t),
\end{align*}
\]

\[
\hat{x}(t) = \sum_{i=1}^{q} \theta_i(z(t))\theta_j(z(t))(A_i - B_i K_j)\dot{x}(t) \\
&\quad + \sum_{i=1}^{q} \sum_{j=1}^{q} \theta_i(z(t))\theta_j(z(t))L_j C_j \varepsilon(t),
\]

where \(\varepsilon(t) = y(t) - \hat{y}(t)\).
\[
\dot{x}(t) = \sum_{i=1}^{n} \sum_{j=1}^{n} \theta_i(z(t)) \theta_j(z(t))((A_i - \Delta B_i K_j)x(t) + \sum_{i=1}^{n} \sum_{j=1}^{n} \theta_i(z(t)) \theta_j(z(t))(A_i - L_i C_j - \Delta B_i K_j)e(t).
\]

Define
\[
\Psi^T(t) = [x^T(t) \quad e^T(t)],
\]
\[
V(\Psi(t)) = x^T(t)Px(t) + e^T(t)Re(t),
\]
where \(P \) and \( R \) are positive-definite symmetric matrices. The time derivative of \( V(\Psi(t)) \) is represented as follows:
\[
\dot{V}(\Psi(t)) = \sum_{i=1}^{n} \sum_{j=1}^{n} \theta_i(z(t)) \theta_j(z(t))\{x^T(t)(G_{ij}^T P + PG_{ji}) \times x(t) + 2x^T(t)P(B_i + \Delta B_i)K_je(t) + 2e^T(t)R \times (A_i - \Delta B_i K_j)x(t) + e^T(t)(H_{ij}^T + R + RH_{ji})e(t)\},
\]
where \( G_{ij} = A_i + \Delta A_i - (B_i + \Delta B_i)K_j \) and \( H_{ij} = A_i - L_i C_j - \Delta B_i K_j \). Since the uncertainties in (12) may be time-varying, it is not easy to manage. For solving these difficulties, consider the following lemma and proposition:

**Lemma 1:** The LMI
\[
\begin{pmatrix}
Q(y) + \rho I & S(y) \\
* & R(y)
\end{pmatrix} < 0
\]
is equivalent to
\[
R(y) < 0, \quad Q(y) - S(y)R(y)^{-1}S(y)^T < -\rho I,
\]
where \(Q(y) = Q(y)^T, \quad R(y) = R^T(y), \quad S(y)\) depend affinely on \(y\).

**Proof:** This proof is a trivial extension of Schur complement of [20].

**Proposition 1:** Using Lemma 1 and Assumption 1, the uncertain terms in (12) are solved as following:
\[
x^T(t)(\Delta A_i^T P + P\Delta A_i)x(t)
\leq \varepsilon_1 x^T(t)E_{ii}^T E_{ii}x(t) + \varepsilon_1^{-1}PD_iD_i^T P x(t).
\]

**Proof:** Using Assumption 1, the following inequality holds:
\[
x^T(t)(\Delta A_i^T P + P\Delta A_i)x(t)
= -\left[ \sqrt{\varepsilon_1^{-1}} D_i^T P x(t) - \sqrt{\varepsilon_1^{-1}} E_{ii} F_i(t) E_{ii} P x(t) \right]^T
\times \left[ \sqrt{\varepsilon_1^{-1}} D_i^T P x(t) - \sqrt{\varepsilon_1^{-1}} E_{ii} F_i(t) E_{ii} P x(t) \right]
+ \varepsilon_1 x^T(t)D_iD_i^T P x(t)
+ \varepsilon_1 x^T(t)E_{ii} F_i(t) F_i(t) E_{ii} x(t)
\leq \varepsilon_1 x^T(t)E_{ii}^T E_{ii} x(t) + \varepsilon_1^{-1}PD_iD_i^T P x(t).
\]

**Remark 1:** By Proposition 1, the remain uncertain terms are represented as follows:
\[
x^T(t)(-K_j^T \Delta B_i^T P - P \Delta B_i K_j)x(t)
+ e(t)^T(-K_j^T \Delta B_i R - R \Delta B_i K_j)x(t)
+ x(t)^T P \Delta B_i K_j e(t) + e(t)^T K_j^T \Delta B_i^T P \times x(t) + x(t)^T \Delta A_i^T R e(t) + e(t)^T R \Delta A_i x(t)
- x(t)^T K_j^T \times \Delta B_i^T R e(t) - e(t)^T R \Delta B_i K_j e(t)
\leq \varepsilon_2^{-1} x^T(t)PD_iD_i^T P x(t)
+ \varepsilon_3 x^T(t)K_j^T E_{ii}^T E_{ii} K_j x(t)
+ \varepsilon_3^{-1} e^T(t)RD_i^T P e(t)
+ \varepsilon_4 x^T(t)PD_iD_i^T P e(t)
+ \varepsilon_4^{-1} x^T(t)K_j^T E_{ii}^T E_{ii} K_j e(t)
+ \varepsilon_5 x^T(t)K_j^T E_{ii}^T E_{ii} K_j e(t)
+ \varepsilon_5^{-1} x^T(t)RD_iD_i^T R e(t)
+ \varepsilon_6 x^T(t)E_{ii}^T E_{ii} x(t)
+ \varepsilon_6^{-1} e^T(t)RD_iD_i^T R e(t)
+ \varepsilon_7 x^T(t)K_j^T E_{ii}^T E_{ii} K_j x(t).
\]

Before proceeding further, recall the following lemma.

**Lemma 2** [15]: For any real matrices \(A_1 = A_1^T, A_2, A_3(t), \) and \(A_4 \) with appropriate dimensions, the following inequality holds:
\[
A_1 + A_2 A_3(t) A_4 + A_4^T A_3^T(t) A_2^T < 0,
\]
where \(A_3(t)\) satisfies \(A_3^T(t)A_3(t) \leq I\) if and only if
\[
A_1 + \left[ \varepsilon_4^{-1} A_4^T \varepsilon_2 \varepsilon_2^{-1} A_4 \right] < 0
\]
for some \(\varepsilon > 0\).

Now, we propose the following LMI results for the asymptotic stability of (7) and (9). The fuzzy control gain \(K_i\) and observer gain \(L_i\) can be solved by the following LMI.

**Theorem 1:** Suppose that the matrices \(P = P^T > 0, \quad R = R^T > 0, \quad M_j, N_j, \) nonsingular matrix \(\hat{P}\) and the scalars \(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6\) are optimal solutions to
\[
\begin{pmatrix}
\Phi_{11} & * & * & * \\
K_j^T B_i^T P & \Phi_{22} & * & * \\
D_i^T & 0 & -\varepsilon_1 I & * \\
K_j^T D_i^T P & 0 & 0 & -\varepsilon_2 I \\
0 & E_2 K_j^T + D_i^T R & 0 & 0 \\
D_i^T & E_2 K_j^T & 0 & 0 \\
0 & D_i^T R & 0 & 0 \\
E_2 K_j & D_i^T R & 0 & 0
\end{pmatrix} < 0
\]
where $\Phi_{11} = A_i^T P + PA_i - K_j B_i^T P - PB_j K_j$, $\Phi_{22} = A_i^T R + RA_i - C_j^T L_j^T R - RL_j C_j$. Then, $(x(t)$ of (7) is globally exponentially stable. The fuzzy gains are given by $K_j = \hat{P}^{-1} M_i$, $i \in I_q$ and fuzzy observer gains are obtained by $L_i = R^{-1} N_i$, $i \in I_q$.

**Proof:** The proof is given in Appendix A.

**Remark 2:** It is notice that (15) is affined with all their respective arguments so that we can use the Theorem 1 to obtain the control and observer gain.

**Remark 3:** In linear system, it is possible to distinguish the conservative problems by selecting the matrix $P$ [4]. However, in nonlinear system, the conservative problems are complex one because the linear independence of the $B_i$ and $PB_i$ are not always so. Therefore, it is necessary to maintain the $\hat{P}$ as nonsingular.

Consider the following observer-based systems which have the another uncertainty $\Delta C_i$:

$$
\dot{x}(t) = \sum_{i=1}^{q} \sum_{j=1}^{q} \theta_i(z(t)) \theta_j(z(t)) (A_i + \Delta A_i - B_i K_j) x(t) \\
Y(t) = \sum_{i=1}^{q} \sum_{j=1}^{q} \theta_i(z(t)) \theta_j(z(t)) (C_i + \Delta C_i) x(t),
$$

where $[\Delta A_i \Delta C_i] = D_i F_i(t) [E_{ii} \ E_{i3}]$ with $F_i^T(t) F_i(t) \leq I$.

In order to estimate the error $e(t)$, we use the full-order observer (4). Then, we obtain the following closed-loop system:

$$
\dot{x}(t) = \sum_{i=1}^{q} \sum_{j=1}^{q} \theta_i(z(t)) \theta_j(z(t)) ((A_i + \Delta A_i) - B_i K_j) x(t) \\
+ \sum_{i=1}^{q} \sum_{j=1}^{q} \theta_i(z(t)) \theta_j(z(t)) B_i K_j e(t),
$$

$$
\dot{e}(t) = \sum_{i=1}^{q} \sum_{j=1}^{q} \theta_i(z(t)) \theta_j(z(t)) ((\Delta A_i - L_j \Delta C_j) x(t)
+ \sum_{i=1}^{q} \sum_{j=1}^{q} \theta_i(z(t)) \theta_j(z(t)) (A_i - L_i C_j) e(t).
$$

Define the Lyapunov functional candidate as

$$
V(\overline{\Psi}(t)) = x^T(t) \overline{P} x(t) + e^T(t) \overline{R} e(t),
$$

where $\overline{\Psi}(t) = [x^T(t) \ \overline{e}^T(t)]$, $\overline{P}$ and $\overline{R}$ are positive-definite symmetric matrices. Considering Assumption 1, Lemma 1 and 2, the time derivative of $V(\overline{\Psi}(t))$ is computed by

$$
\dot{V}(\overline{\Psi}(t)) = \sum_{i=1}^{q} \sum_{j=1}^{q} \theta_i(z(t)) \theta_j(z(t)) (x^T(t) (\overline{G}_{ij}^T \overline{P} + \overline{P} \overline{G}_{ij}) \\
\times x(t) + 2 x^T(t) \overline{P} B_j K_j e(t) + 2 e^T(t) \overline{R} (\Delta A_i
- L_j \Delta C_j) x(t) + e^T(t) (\overline{H}_{ij}^T \overline{R} + \overline{R} \overline{H}_{ij}) e(t)),
$$

where $\overline{G}_{ij} = A_j + \Delta A_j - B_j K_j$ and $\overline{H}_{ij} = A_i - L_i C_j$. By Proposition 1, the uncertain terms in (21) are solved as follows:

$$
x^T(t) (\Delta A_i^T \overline{P} - \overline{P} \Delta A_i) x(t) \\
+ x^T(t) (\overline{A}_i - L_j \Delta C_j)^T \overline{R} e(t) \\
+ e^T(t) \overline{R} (\overline{A}_i - L_j \Delta C_j) x(t) \\
\leq (\epsilon_1 + \epsilon_2) x^T(t) E_{ii} E_{ii} x(t) \\
+ \epsilon_3 x^T(t) E_{3i} E_{3i} x(t) \\
+ \epsilon_4 x^T(t) D_i^T D_i \overline{P} x(t) \\
+ \epsilon_5 x^T(t) \overline{R} D_i D_i^T L_j \overline{R} e(t) \\
+ \epsilon_5 x^T(t) \overline{R} L_j D_i D_i^T L_j \overline{R} e(t),
$$

where $\epsilon_1$, $\epsilon_2$ and $\epsilon_3$ are positive constants. The new results are summarized as follows:

**Theorem 2:** Suppose that the matrices $\overline{P} = \overline{P}^T > 0$, $\overline{R} = \overline{R}^T > 0$, $M_i$, $N_i$, nonsingular matrix $\hat{P}$ and the scalars $\epsilon_1$, $\epsilon_2$, $\epsilon_3$ are optimal solutions to

$$
\begin{bmatrix}
\Phi_{11} & * & * & * \\
K_{i} \hat{P}^T & * & * & * \\
D_i^T \overline{P} & 0 & -\epsilon_2 I & * \\
0 & D_i^T \overline{R} & 0 & -\epsilon_3 I \\
0 & D_i^T L_j^T & 0 & 0 & -\epsilon_5 I
\end{bmatrix} < 0,
$$

where $\Phi_{11} = A_i^T P + P A_i - K_j B_i^T P - P B_j K_j$, $\Phi_{22} = A_i^T R + R A_i - C_j^T L_j^T R - R L_j C_j$. Then, $x(t)$ of (17) is globally exponentially stable. The fuzzy gains are given by $K_j = \hat{P}^{-1} M_i$, $i \in I_q$ and fuzzy observer gains are obtained by $L_i = R^{-1} N_i$, $i \in I_q$.

**Proof:** The proof is given in Appendix B.
4. SYSTEM MODELLING AND SIMULATION RESULTS

4.1. Wind power system and its fuzzy modelling

In this section, we introduce the VSWP system and its fuzzy modelling. A three bladed horizontal-axis 60KW wind turbine is considered and Fig. 1 is the schematic diagram of a concerned system. The dynamics of the VSWP system can be represented as following [16,17]:

\[
Q_A - A = J_\omega \ddot{\omega}_t, \quad Q - Q_E = J_G \dot{\omega}_G,
\]
\[
Q = Q_0 + Q_G = K_s \int (\omega_t - \omega_g) \, dt + B_s (\omega_t - \omega_g),
\]
(25)

where \(Q_t\) is the input wind torque, \(Q_E\) is the generator torque, \(\omega_t\) is the turbine angular velocities, \(\omega_g\) is the generator angular velocities, \(K_s\) is shaft compliance, \(B_s\) is shaft damping, \(J_t\) is turbine inertia, \(J_G\) is generator inertia, and \(Q\) is shaft torque. The power produced by the wind is expressed as [18]

\[
P_T(w, \lambda) = \frac{1}{2} \rho R^2 C_p(\lambda) w^3,
\]
(26)

where \(\rho\) is air density, \(R\) is blade length, \(w\) is wind velocity, and \(C_p\) denotes power coefficient of the wind turbine. It is presented as a nonlinear function of the tip speed ratio \(\lambda\)

\[
\lambda = \frac{R \omega_t}{w}.
\]
(27)

Fig. 2 shows the trajectory of the VSWP system with \(x(0)=[10 \quad -10 \quad 10]^T\).

In order to construct the fuzzy model for the VSWP system, it is necessary to analyze the variable torque coefficient function \(C_p(\lambda)\). This specific function plays an important role of converting kinetic energy of the moving air to mechanical torque \(Q_t\). For optimum energy capture, the wind turbine must operate at the point of maximum power coefficient \(C_{p\text{max}}\) when \(\lambda=\lambda_{\text{opt}}\) [18,22,23]. In the view of above equations (26) and (27), we can obtain the following expression of the aerodynamic torque:

\[
Q_A = \frac{1}{2} \rho AR^2 C_p(\lambda) w^3,
\]
(28)

where \(C_p(\lambda)\) is variable torque coefficient function

\[
C_p(\lambda) = \lambda C_q(\lambda).
\]
(29)

From (28) and (29), we know that \(Q_A\) has a nonlinear function of two variables \((w, \omega_t)\). Denote \(x^T=[x_1 \quad x_2 \quad x_3]=[\omega_t \quad \omega_g \quad Q_t]\), then the dynamics of the VSWP system can be represented as the following equations:

\[
\dot{x}_1 = \frac{\gamma - B_s}{J_T} x_1 + \frac{B_s}{J_T} x_2 + \frac{1}{J_T} x_3,
\]
(30)
\[
\dot{x}_2 = \frac{B_s}{J_G} x_1 - \frac{B_s}{J_G} x_2 + \frac{1}{J_G} x_3 - \frac{1}{J_G} u,
\]
(31)
\[
\dot{x}_3 = K_s x_1 - K_s x_2,
\]
(32)

where \(\gamma = \frac{1}{2} \rho AR^2 C_p(\lambda)\) and in order to construct the T-S fuzzy model for VSWP system, the nonlinear term has to be represented as convex combination of appropriate vertices. Assume \(x_i \in [M_1 \ M_2]\) and consider the following equations:

\[
\eta_i(x_i(t)) = \Gamma_i^1 (x_i(t)) x_i(t) + \Gamma_i^2 (x_i(t)) x_i(t),
\]
(33)

\[
1 = \Gamma_i^1 (x_i(t)) + \Gamma_i^2 (x_i(t)),
\]
(34)

where

\[
\Gamma_i^1 = -x_i + D_i, \quad \Gamma_i^2 = \frac{x_i - D_i}{D_j - M_i}.
\]
(35)

Then, the identified T-S fuzzy rules of the VSWP system are represented as follows:

\[
R_1^1: \text{IF } x_1(t) \text{ is } \Gamma_1^1, \text{ THEN } \tilde{x}(t) = A_1 x(t) + B_1 u(t),
\]
\[
R_2^1: \text{IF } x_1(t) \text{ is } \Gamma_2^1, \text{ THEN } \tilde{x}(t) = A_2 x(t) + B_2 u(t),
\]

where

\[
\begin{align*}
A_1 &= \begin{bmatrix} y_1 - B_s & B_s & 1 \\ B_s & -B_s & 1 \\ 0 & K_s & 0 \end{bmatrix},
A_2 &= \begin{bmatrix} y_2 - B_s & B_s & 1 \\ B_s & -B_s & 1 \\ 0 & K_s & 0 \end{bmatrix},
B_1 &= B_2 = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix}^T.
\end{align*}
\]
4.2. Simulation result
This section presents simulation result of the VSWP system. The nominal values of system parameters are presented as Table 1.

During the simulation time, all system parameters are randomly varied within the bounds of 10% of their nominal values - $J_T$ and $J_G$. Applying Theorem 1 yields the total fuzzy system control gain matrices, for the sampling period $T=0.1s$ as

$$K_1 = [-120.6573, -48.9923, -8.5947],$$
$$K_2 = [-89.9836, -39.6006, -10.3913].$$

Utilizing Theorem 1 yields the observer gain matrix

$$L_1 = [78.7233, 30.0912, -3.0913],$$
$$L_2 = [-71.9981, 22.6239, 0.1323].$$

Table 1. Nominal parameters of the VSWP system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_T$</td>
<td>40000 kg·m²</td>
</tr>
<tr>
<td>$J_G$</td>
<td>65 kg·m²</td>
</tr>
<tr>
<td>$K_S$</td>
<td>100 N·ms/rad</td>
</tr>
<tr>
<td>$B_S$</td>
<td>1800 N·ms/rad</td>
</tr>
<tr>
<td>$R$</td>
<td>35 m</td>
</tr>
<tr>
<td>$p'$</td>
<td>1.225 kg/m³</td>
</tr>
</tbody>
</table>

As shown in Figs. 3-5, all system trajectories converge to zero despite of the system uncertainties. The simulation results show that the T–S fuzzy control through fuzzy observer is robust against norm-bounded parametric uncertainties.

5. CONCLUSIONS

In this paper, the observer-based robust fuzzy control has been proposed. Our main contribution consists in showing that the complex nonlinear control problem can be transformed to find the fuzzy gains stabilizing the whole control systems. Also, the robust stability condition has been successfully incorporated in the proposed method. Since the proposed conditions are LMI-based ones, they can be easily extended to other control problems (output feedback control, decentralized control, $H_{\infty}$ control, etc.). The simulation results for VSWP system have demonstrated that it is possible to obtain the excellence performance through the proposed method.

APPENDIX A

To summarize the equations (12)-(14), we have

$$\dot{V}(\Psi(t)) = \sum_{i=1}^{q} \sum_{j=1}^{q} \bar{\theta}_i(z(t))\bar{\theta}_j(z(t))\left((x^T(t)(A_i^T P + PA_i - K_jB_i^T P - PB_iK_j)x(t) + e(t)^T (A_i^T R + RA_i)
- C_j^T \times L_j R - RL_j C_j)e(t) + 2x(t)^T PB_iK_j e(t)
+ (\varepsilon_1 + \varepsilon_3)x^T(t)e_i^T e_i + (\varepsilon_3 + \varepsilon_4)e^T(t)K_j
\times E_2^T E_2 K_j e(t) + (\varepsilon_3 + \varepsilon_4)e^T(t)RD_i^T D_i
\times e(t) + \varepsilon_1 x^T(t)(PD_iD_i^T P x(t) + \varepsilon_3 x^T(t))RD_i^T R
\times e(t) + \varepsilon_0 x^T(t)K_j E_2^T E_2 K_j e(t)\right)$$

$$= \sum_{i=1}^{q} \sum_{j=1}^{q} \bar{\theta}_i(z(t))\bar{\theta}_j(z(t))\left((\Phi_{11}^{T} \Phi_{12}^{T} \Phi_{12}^{T} \Phi_{22}^{T})\left[\begin{array}{c} \Phi_{11} \\ \Phi_{12} \\ \Phi_{21} \\ \Phi_{22} \end{array}\right] \right)\times e(t) + (\varepsilon_3 + \varepsilon_4)e^T(t)K_j E_2^T E_2 K_j e(t)$$
The system (21) is represented as follows:

\[
\begin{align*}
\dot{X}_{11} &= B_KJ_i \\
\dot{X}_{22} &= 0 \\
\end{align*}
\]

By Lemma 1, the LMI condition (15) implies

\[
\begin{bmatrix}
D_i P & 0 \\
0 & K_j E_{21}^T + RD_i \\
0 & K_j E_{21}^T + RD_i \\
0 & RD_i \\
0 & D_i^T R \\
E_{21} K_j & D_i^T R \\
\end{bmatrix}
\preceq -\rho I.
\]

The matrix is equal to the matrix \( \Phi_q \) with \( P = I \) and \( L = R^{-1} L \), and the system convergence rate is \([\rho/(2 \cdot [\lambda_{\text{max}}(P), \lambda_{\text{max}}(R)])].\) By (12), (13), and above equation we have

\[
\begin{align*}
\min[1, \lambda_{\text{max}}(R)] \|\Psi(t)\|^2 \\
\leq V(\Psi(t)) \leq \max[1, \lambda_{\text{max}}(R)] \|\Psi(t)\|^2
\end{align*}
\]

and

\[
\dot{V}(\Psi(t)) \leq -\rho \|\Psi(t)\|^2.
\]

**APPENDIX B**

The system (21) is represented as follows:

\[
\begin{align*}
\dot{X}_{11} &= B_KJ_i \\
\dot{X}_{22} &= 0 \\
\end{align*}
\]

**REFERENCES**


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